Rail Cube technical notes

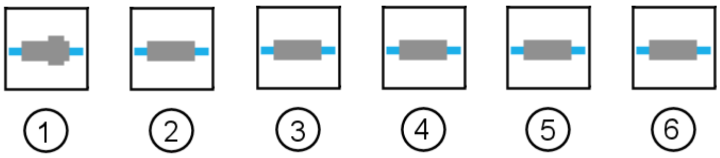
Description

The Rail Cube is a variant of Rubik's Cube. You play here with rail tracks, rail cars and a loco. The purpose of the puzzle is to build track networks to connect these vehicles in various ways.

Content

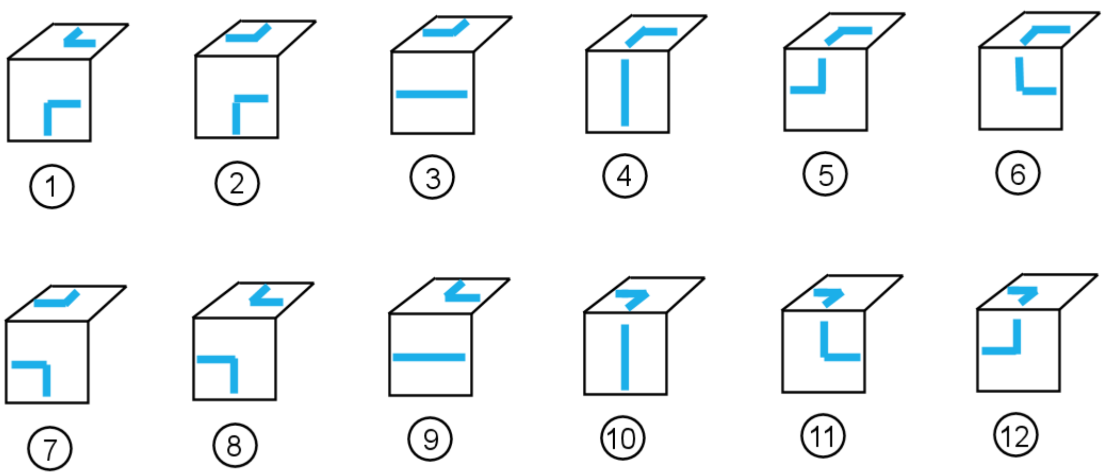
A 3x3x3 cube contains 6 centers, 12 edges and 8 corners. A center has 1 facet, an edge has 2 facets and a corner has 3 facets. In total we have 54 facets, each one containing a piece of track, either straight or curved.

Below are the 6 centers containing 6 vehicles placed on a straight track: 1- loco, 2-6 cars in different colors (green, blue, orange, red, violet).



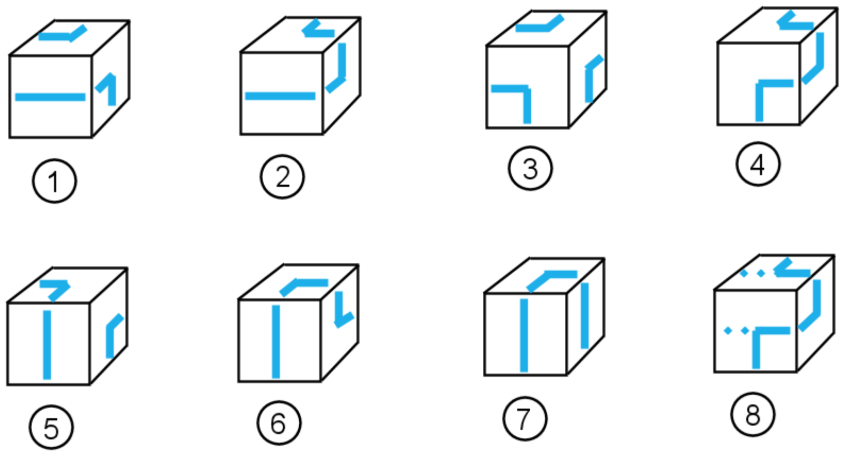
Each center has 2 exits to the edges. In total 12 exits.

Below are the 12 edges. Some edges are identic (1-7 and 6-12). They can be swapped as needed. These edges are ornated with some trees for easy recognition. Some edges are symetric (2,5,8,11). They can be flipped as needed.



We may classify the edges as follows: with exits to centers (1,2,3,4,7,8,9,10) and without (5,6,11,12), with a total of 4 exits (1,2,3,7,8,9) and with 2 exits (4,5,6,10,11,12). In total there are 12 exits to centers and 24 exits to corners.

Next are the 8 corners. There are two identic corners (4-8) which can be swapped as needed, with the specification that corner 8 is in fact a turnout with multiple exits. Two of these exits are red-signaled, showing that they are not a part of the full solution (see below), but otherwise they can be used. One corner is symmetric (3) and it can be freely rotated. Corner 7 is ornated with a railway platform. It serves just as a reference point on the network and it has no other particular meaning.



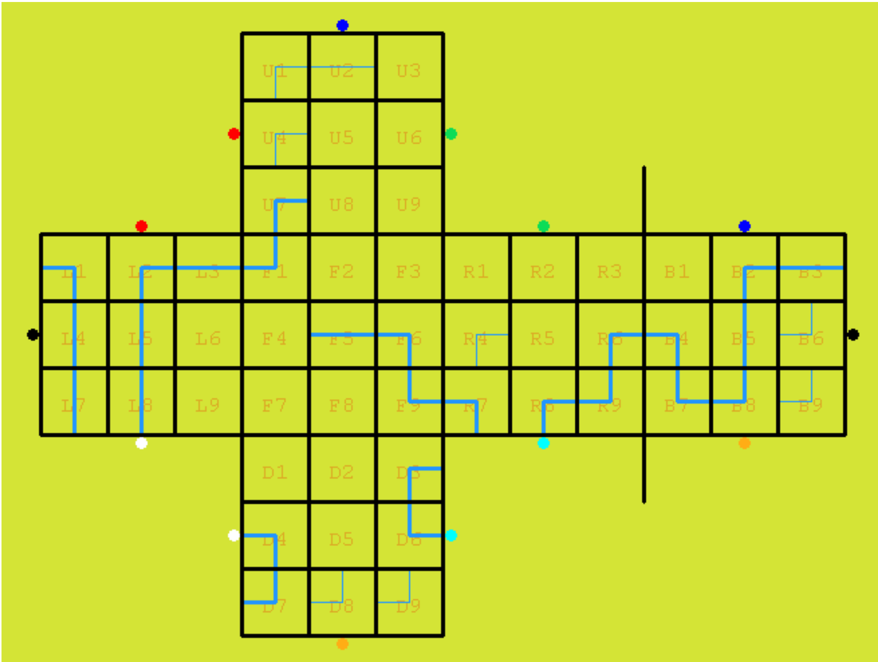
There are corners with 6 exits (3), with 4 exits (1,5) and with 2 exits (2,4,6,7,8). In total there are 24 exits to the edges. The two red-signaled exits are not counted here.

We see that the number of exits in pairs "centers-edges" and "edges-corners" do match. It means that, theoretically, a full network loop using all 54 individual tracks is possible. Such a loop will connect all 6 vehicles on the cube and it is called a <u>full solution</u>. Any track network that uses all 54 individual tracks, in a loop or not, single or multiple lines, but with the condition that any individual track line is accessible to at least one vehicle, is called a <u>partial solution</u>.

Besides of these solutions which are hard to find, an easy task on this cube is to connect all the 6 vehicles (eventually in a particular order) using any number of individual tracks. This can be accomplished in many ways.

Computing solutions

A computer program was created to find full and partial solutions. Basically, it generates sequences of cube pieces (centers, edges and corners) taken from previous tables and placed on a cube layout as in the next figure. The starting point is center F5 on the front face. A continous track line is build an followed, using recursion, until a particular goal is reached. In certain conditions the program is able to search backward from the F5 center or to initiate an additional search from another center piece.



In the next sections we will summarize several interesting solutions. Using the image layouts provided below someone can recreate the track network on his cube. Let see briefly how these solutions can be applied on a real cube.

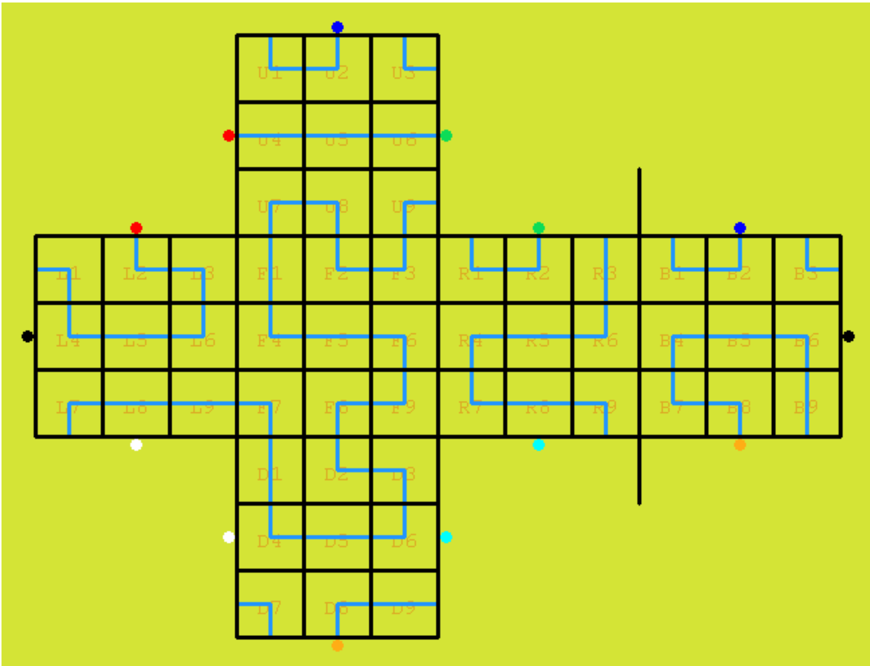
Applying solutions

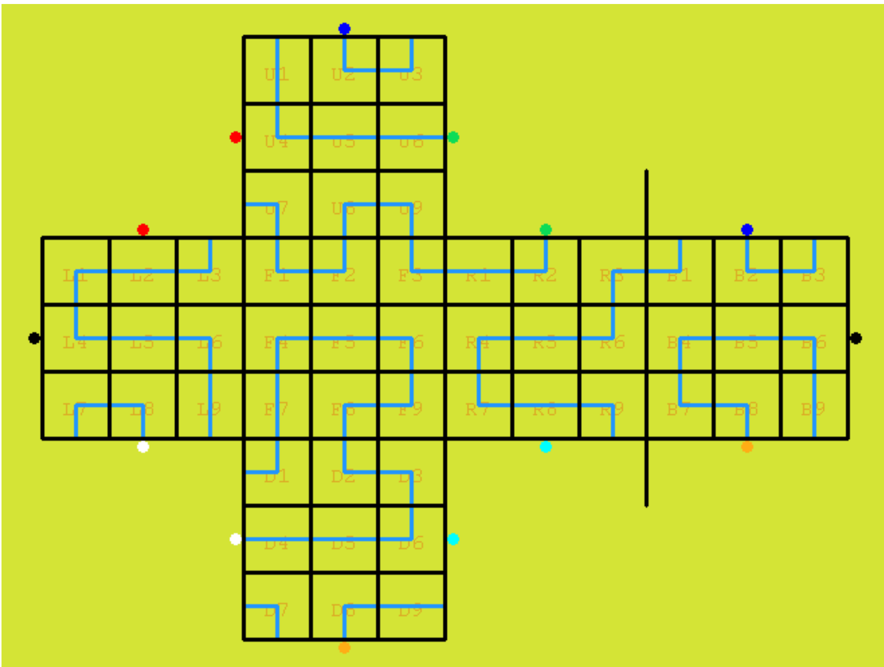
It is known that on a Rubik's Cube there are some constrains regarding positioning and orientation of pieces. For example, one single center cannot be rotated 90 degrees, one single edge cannot be flipped, one single corner cannot be rotated, etc. On Rail Cube these constrains are overcomed, because there are some identic pieces and symmetric pieces, as discussed on previous sections. So, any solution found on paper can be applied on the cube. For example, one single center can be rotated 90 degrees by swapping two identic corners and two identic edges. Or, one single corner can be rotated by rotating also in opposite direction the symmetric corner number 3.

Also, if we look at an image solution in a mirror (by reflection), we obtain a mirror solution. It is interesting that always a mirror solution can be applied on a real cube along with the normal solution, because each cube piece (edge or corner) has its own mirror counterpart. A normal solution and its mirror solution are not really equivalent, because left face and right face are swapped, thus the order of vehicles on the track line is different.

Full solutions

Except cube rotations and reflections, there are two distinct full solutions. First one is considered the initial state of the cube. The main difference between these solutions is that, in the first one, two opposite centers (vehicles) are consecutively connected. There are no full solutions using red-signaled exits of the turnout.





Partial solutions

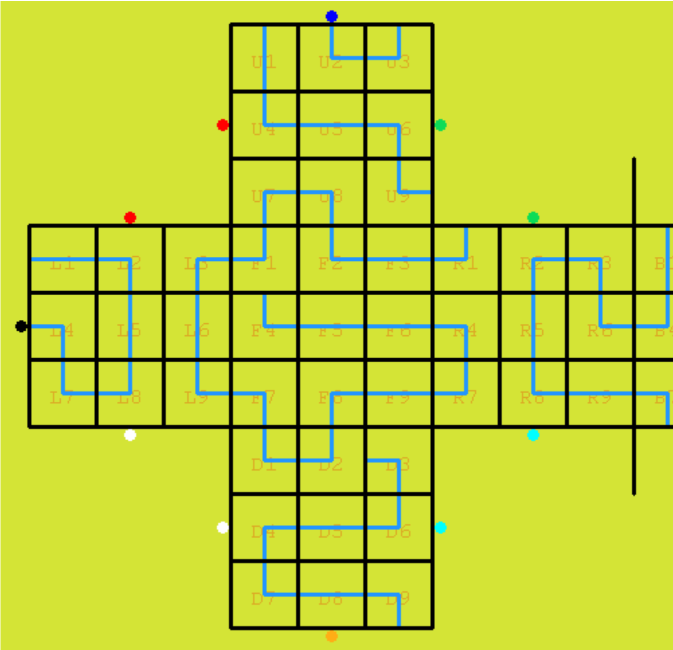
The number of partial solutions is very large, because of the definition we employed. This number is even larger if using all turnout exits. The unused turnout exits are not marked on the images thus, the position of the turnout in the layout is, sometimes, not so obvious. Below we summarize some catagories of partial solutions.

- single non-loop line

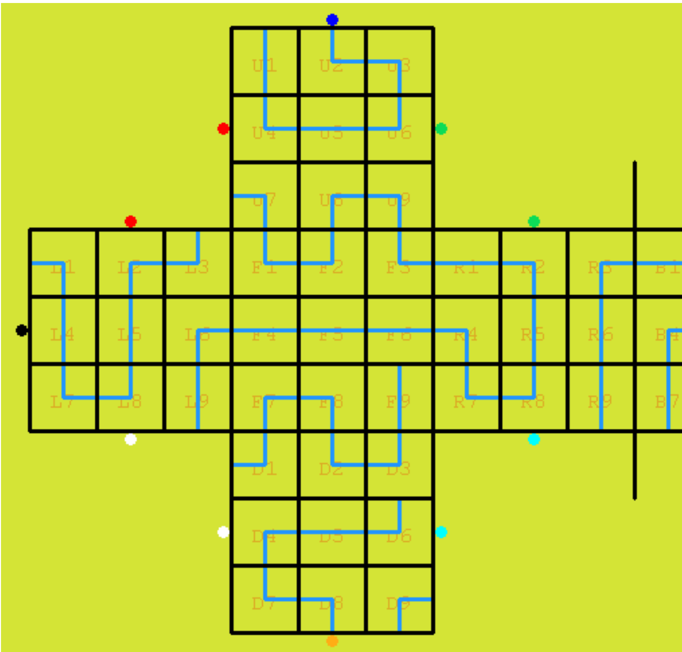
An interesting goal on Rail Cube is to find a partial solution with a single non-loop track line, where all vehicles are connected in a particular order (ex. “rainbow arrangement”: loco - red car - orange car – green car – blue car – violet car). Of course, some arrangements can be obtained from others by selecting a different front face vehicle and/or right face vehicle before building the track line, or by selecting the mirror solution. Excluding such “tricks” there are, however, arrangements that needs distinct partial solutions to be found.

Next we try to encode such an arrangement. Let the first vehicle on the track line and the opposite one be encoded by digit “1”. The second vehicle on the track line, if not already encoded, is encoded by digit “2” together with its opposite mate. Remaining two vehicles are encoded by digit “3”. So, the rainbow arrangement exemplified above is encoded as “122313”. We were not able to find the arrangements “112233”, “122331”, “112332”, “‘122133”, ”121323” or “123312”. Any other arrangements are possible. Some interesting arrangements are “123123”, “123321”, already listed among solutions in this section.

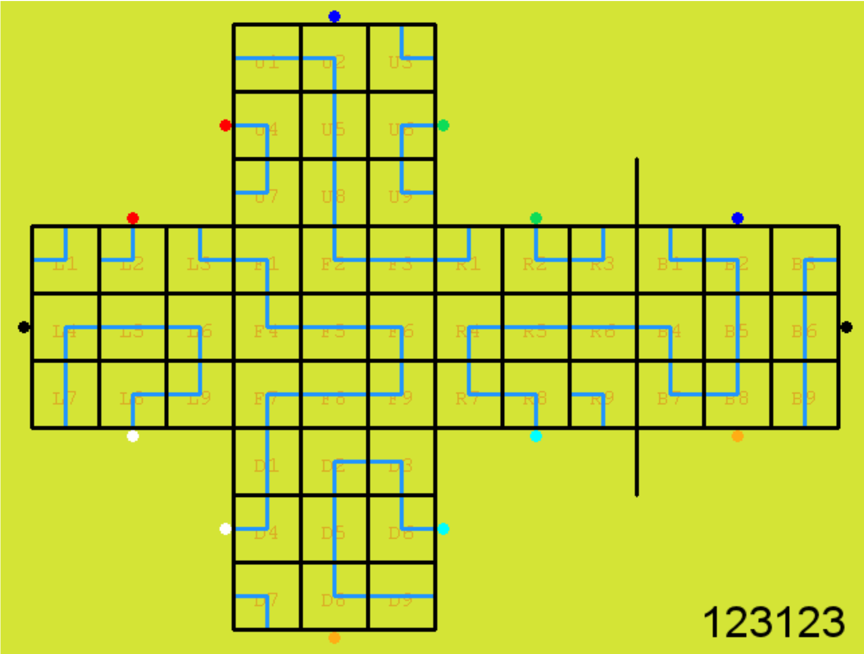
Note that some arrangements can be obtained starting from full solutions and rotating one single corner. For example, starting with the first full solution, by rotating corner number 5 counter-clockwise, we obtain “112323”. Of course, by following the track line in a backward direction we might say that we have obtained a “new” arrangement. For example, the rainbow arrangement “122313” becomes “121332”.

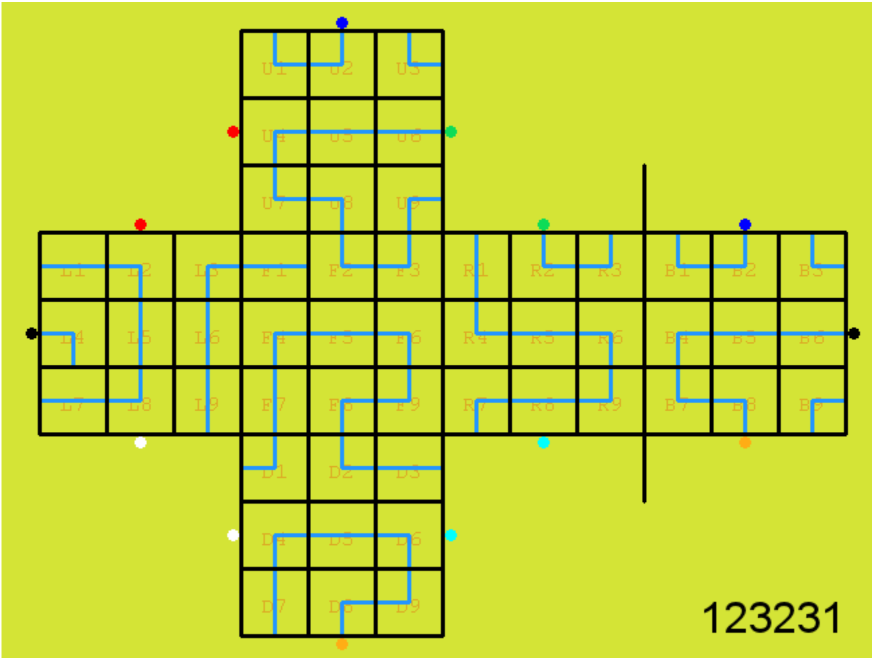


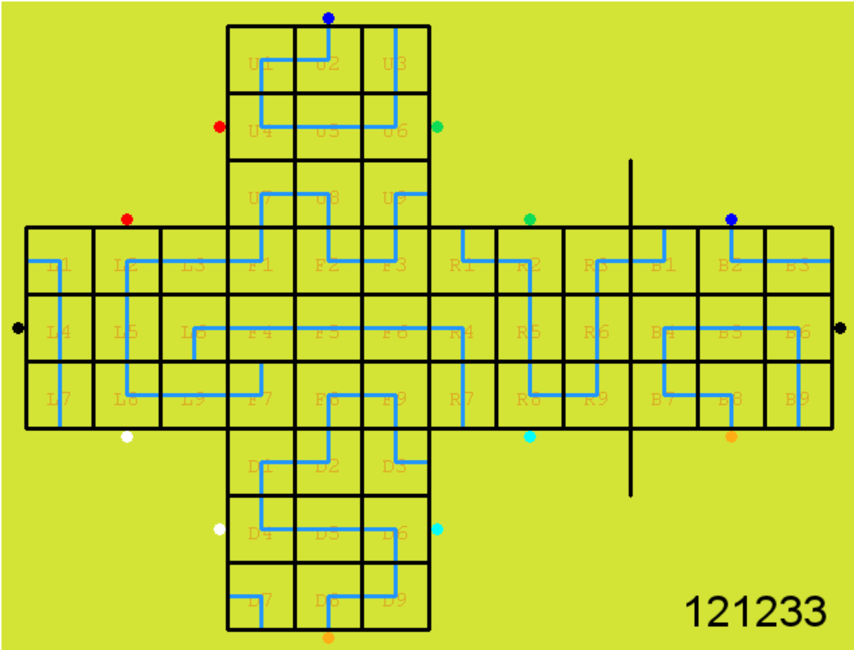
123132

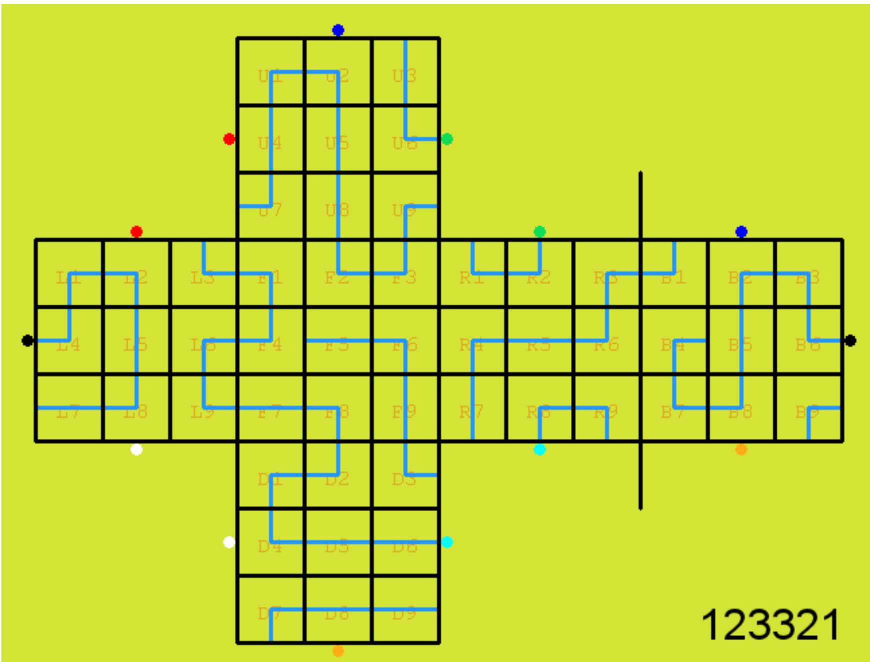


122313


123123

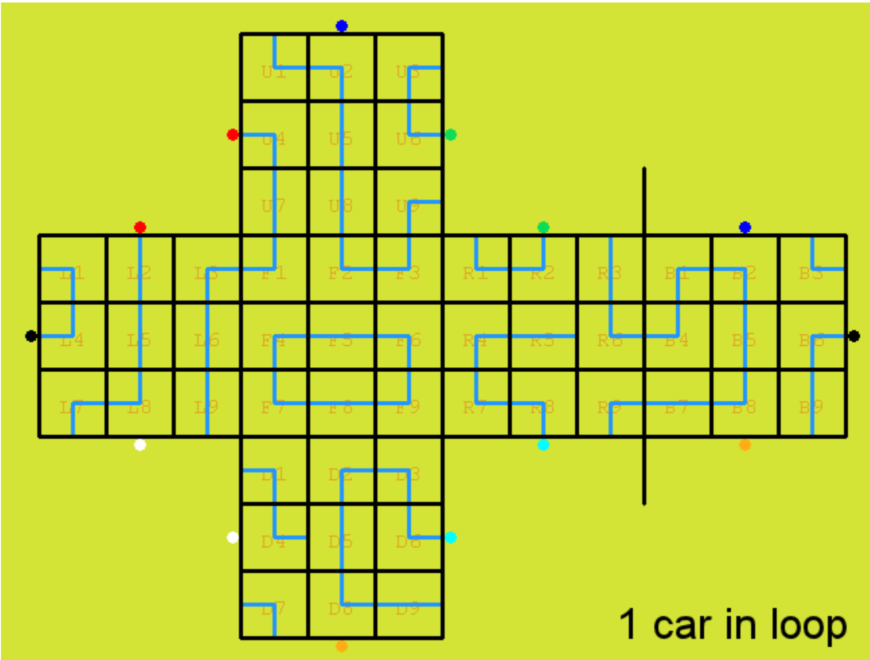

123231

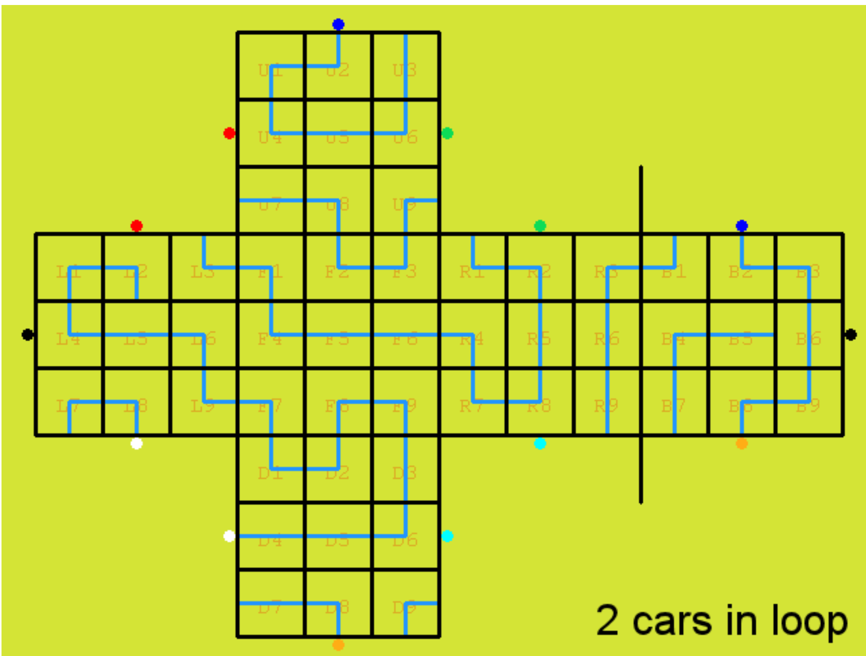

121233

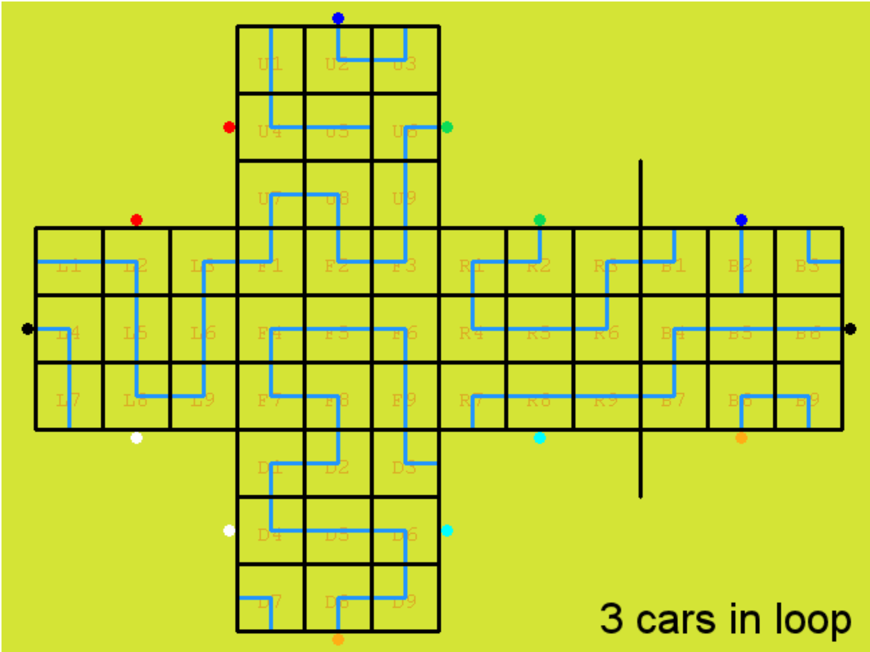

123321

- one loop line and one non-loop line

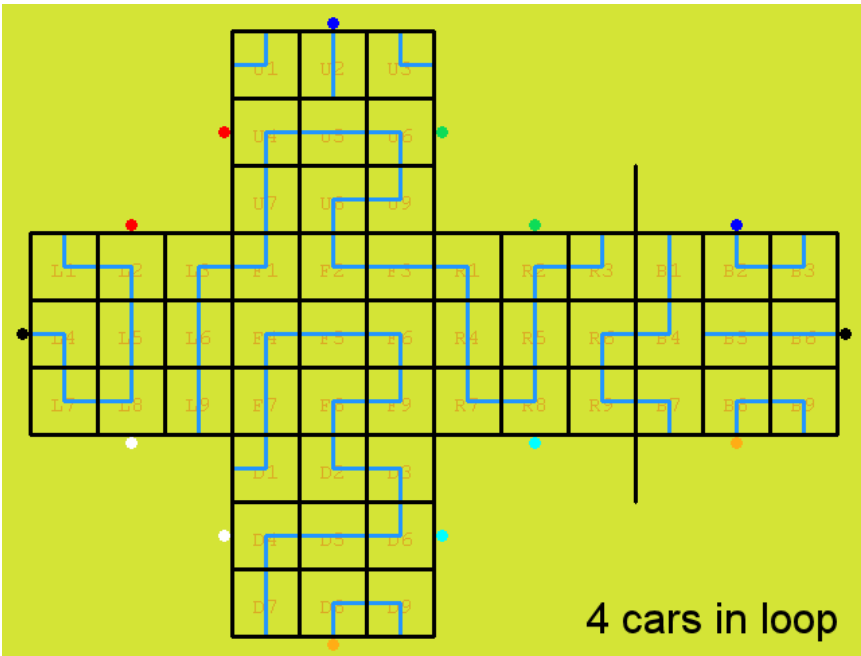
We did not find two non-intersecting loops on the cube and, so, we are bounded to be satisfied with this type of network. Anyway, there are several variants in which the loop contains 1, 2, 3 or 4 vehicles and the rest of them are on the non-loop line.







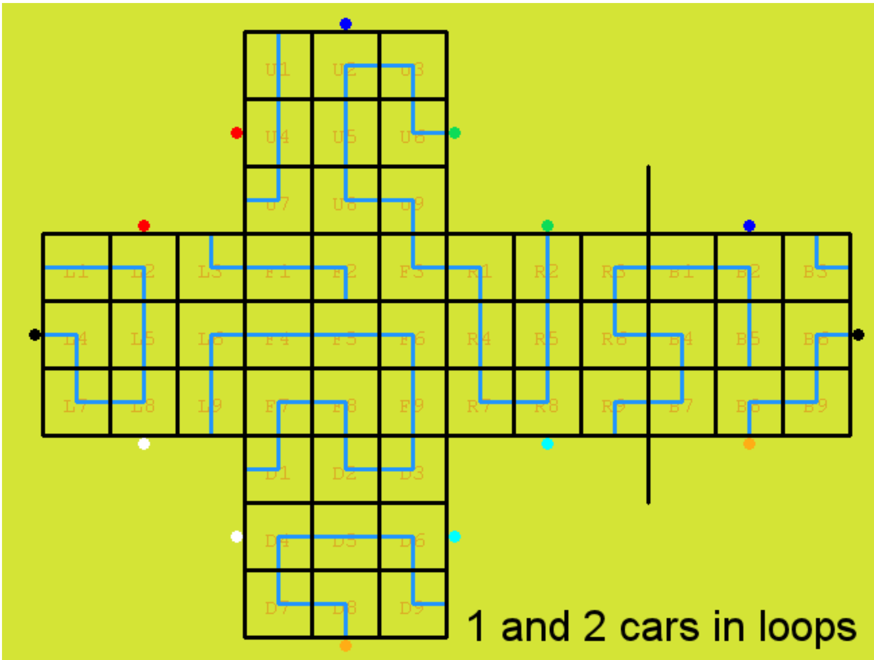
3 cars in loop

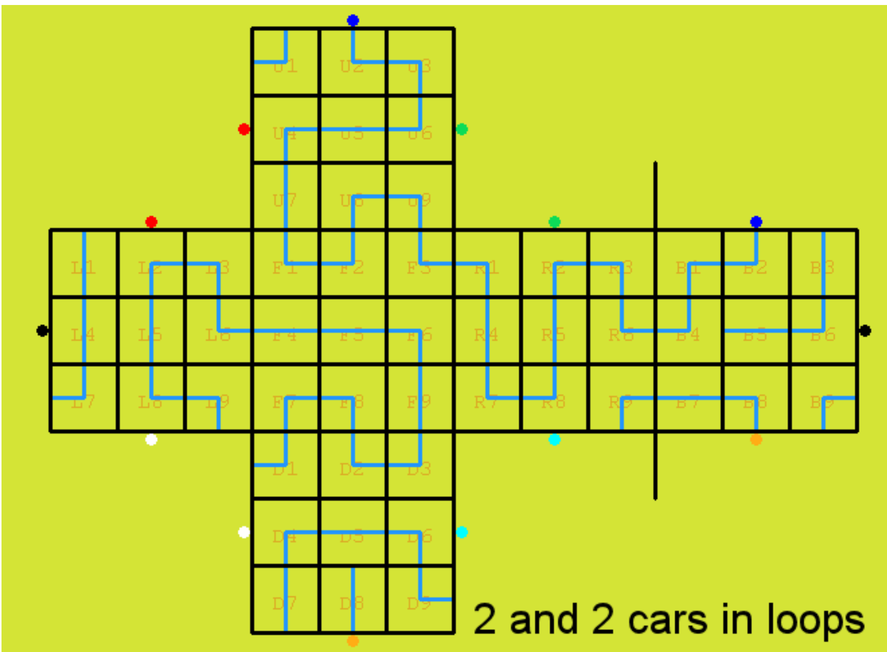


4 cars in loop

- two loop lines and one non-loop line

This is an extension of the previous case. How many vehicles can be placed on these two loops? The answer is 1-2 or 2-2, the rest of them being on the non-loop line.





- turnout branch lines

We are still searching for a network of connected lines using all 4 exits of the turnout. On each branch line at least one vehicle should be placed. This kind of network is particularly interesting in creating shunting puzzles.

Train shunting puzzles

Shunting puzzles consist of a specific track layout, a set of initial conditions, a defined goal and rules which must be obeyed while performing the shunting operations. For example, in the image below, use the loco to (imaginary) move the orange car next to violet car on the upper face, then at the end, the violet car and the loco must return to their initial position.

Final remark

Even assisted by a computer program we did not try an exhaustive search of all partial solutions. We focused just on some particular cases. For sure, there are many other interesting network layouts waiting to be discovered by cube lovers.

22 september 2010,

Stefan Berinde
sberinde@math.ubbcluj.ro